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ENERGETIC VARIANT OF THE MODEL OF RHEOLOGICAL DEFORMATION AND DESTRUCTION OF METALS UNDER A JOINT ACTION OF STATIC AND CYCLIC LOADS

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A uniaxial phenomenological energetic model for description of inelastic deformation and destruction of metals under a joint action of static and cyclic loads is proposed. The amplitude value of the cyclic component of stress in experiments was less than 10% *of static loading. The model proposed was carefully verified in experiments with the* ÉP742 *alloy for* $T = 650$ *and* 750°C. *The numerical and experimental data are in good agreement.*

1. The energetic [1, 2] and adjoining thermodynamic [3, 4] approaches for description of inelastic rheological deformation and destruction of metals under conditions of unsteady loading yield good results, and the advisability of using these approaches in computational practice is unquestionable. The goal of the present work is to generalize the approach [2] for description of a class of events that occur in a material under a joint action of static σ_0 and cyclic loads with an amplitude value of the cyclic component σ_{a_0}. We confine ourselves to consideration of the so-called multicycle loading at frequency $f > 10$ Hz and amplitude coefficient $A = \sigma_0/\sigma_{a_0}$ that does not exceed some critical value $A_{\rm cr}$ of the order of 0.10–0.15 (in contrast to few-cycle loading at frequency $f > 10$ Hz and the number of cycles needed for destruction less than 10^4). In the case considered, the cyclic load leads to two basic effects [5]: 1) acceleration (or even initiation) of the creep process under a given static stress σ_0; 2) reduction of the accumulated inelastic deformation at the moment of destruction as compared to that under purely static loading. This process is called the *cyclic* creep [5] or the *vibrocreep* (for an amplitude coefficient of 0.01–0.03) [6]. It is usually impossible to describe these phenomena within the framework of traditional classical approaches, or phenomenological creep [7], or fatigue under an asymmetric cycle [8]. From the analysis of papers on cyclic creep [6, 9–11], the following approaches can be conventionally distinguished at the phenomenological level.

A. Introduction of reduced stress equal to static stress such that the longevity in the static creep regime coincides with the longevity in the cyclic creep regime. This approach postulates the similarity of the curves of static and cyclic creep, which is one of its drawbacks. In addition, under unsteady loading regimes, these theories yield large errors both for the static and cyclic components.

B. Description of creep under cyclically varying stress. In this case, the behavior of deformation in each cycle is considered. The drawback of this approach is the neglect of fatigue-induced damage and the absence of the loading frequency in the governing equations.

C. Phenomenological models based on the hypothesis of additivity of the fatigue-induced damage parameters and static creep and on the principle of linear summation of the damages:

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$$\int_0^{t_*} \frac{d\tau}{\tau_*(\sigma_0)} + \int_0^{N_*} \frac{dR}{R(\sigma_{a_0,f})} = 1, \tag{1.1}$$

where $\tau_*(\sigma_0)$ is the time of destruction due to creep under a given static loading σ_0 and $R(\sigma_{a_0}, f)$ is the number of cycles prior to destruction for a given amplitude of the cyclic component σ_{a_0} and loading frequency f. Experimental studies show that relation (1.1) is valid only under sequential loading by static and cyclic loads with a moderate gradient. In the rest of the cases, the values of the expression in the left side of equality (1.1) can be significantly greater or smaller than unity. Numerous attempts at creating a universal principle of nonlinear summation of the damages have not been successful.

2. The basic model of solving the problem posed in the present paper is the model proposed by Radchenko [2] for quasi-static loading:

$$\varepsilon = e + e^p + p, \quad \dot{e} = \frac{\dot{\sigma}}{E}, \quad \dot{e}^p = \chi S'(\sigma)\dot{\sigma}, \quad p = u + v + w, \quad u(t) = \sum_{k=1}^{K} u_k(t),$$

$$\dot{u}_k(t) = \lambda_k\Big[a_k\Big(\frac{\sigma(t)}{\sigma_*}\Big)^{n_1} - u_k(t)\Big], \quad v(t) = \sum_{k=1}^{K} v_k(t), \tag{2.1}$$

$$\dot{v}_k = \begin{cases} \lambda_k\Big[b_k\Big(\dfrac{\sigma(t)}{\sigma_*}\Big)^{n_1} - v_k(t)\Big], & b_k\Big(\dfrac{\sigma(t)}{\sigma_*}\Big)^{n_1} > v_k(t), \\ 0, & b_k\Big(\dfrac{\sigma(t)}{\sigma_*}\Big)^{n_1} \leqslant v_k(t), \end{cases} \qquad \dot{w}(t) = c\Big(\frac{\sigma(t)}{\sigma_*}\Big)^m;$$

$$\sigma = \sigma_0(1 + \omega); \tag{2.2}$$

$$\dot{\omega} = \gamma\sigma\dot{e}^p + \alpha\sigma\dot{p}. \tag{2.3}$$

Here ε is the total strain, e and e^p are the elastic and plastic strains, p is the creep-induced strain, u, v, and w are the viscoelastic, viscoplastic, and viscous components of p, σ_0 and σ are the nominal and actual stresses, E is the Young modulus, λ_k, a_k, b_k, c, n, m, and σ_* are rheological constants of the material that describe the first and second stages of creep and the reversible part of the creep deformation, ω is the damage parameter, assumed to be proportional to a linear combination of the work of the actual stress on the creep deformation and the work on plastic deformation, and α and γ are the material parameters that control the loss of strength. We have $\chi = 1$ for $\sigma(t) > \sigma(\tau)$ $(0 \leqslant \tau < t)$ and $\chi = 0$ if it is possible to find a time τ such that $\sigma(t) \leqslant \sigma(\tau)$.

In the general case, we have $\gamma = \gamma(e^p)$ and $\alpha = \alpha(\sigma_0)$, and we can use a power-law approximation for them [2]:

$$\gamma = \gamma_1(e^p)^{m_2}, \qquad \alpha = \alpha_1(\sigma_0)^{m_1}. \tag{2.4}$$

For a number of materials, in special cases, we have $\gamma = \text{const}$ and $\alpha = \text{const}$ [2]. The function $S(\sigma)$, which describes the plasticity strain, has the form

$$S(\sigma) = a(\sigma - \sigma_+)^n, \tag{2.5}$$

where a and n are constants and σ_+ is the limit of proportionality.

3. To generalize model (2.1)–(2.5) to the case of a joint action of quasi-cyclic and cyclic loads, we introduce one more term related to irreversible processes under cyclic loading into the damage parameter. We adopt a hypothesis that the fatigue-induced damage during one cycle of loading is proportional to the applied elastic work of the actual amplitude stress during one cycle at constant σ_0, σ_{a_0}, and f. Then relation (2.1) takes the form

$$\dot{\omega} = \gamma(e^p)\sigma\dot{e}^p + \alpha(\sigma_0)\sigma\dot{p} + g_1(\sigma_0, \sigma_{a_0}, f)\,\frac{\sigma_a^2}{2E}\,\dot{N}, \tag{3.1}$$

where σ_a is the actual value of the amplitude stress

$$\sigma_a = \sigma_{a_0}(1+\omega), \tag{3.2}$$

$g_1(\sigma_0, \sigma_{a_0}, f)$ is a function determined experimentally, and N is the number of loading cycles.

Thus, the complete system of equations for inelastic deformation under a joint action of quasi-static and cyclic loads consists of relations (2.1), (2.2), (3.1), and (3.2).

4. To obtain the destruction criterion, similarly to [2], we use thermodynamic considerations according to which the destruction of a material occurs when the density of internal energy reaches a critical value. The theoretical and experimental research [12] allows us to assume that the critical value of the internal-energy density is independent of the loading process and is a constant of the material.

The internal energy U_+ accumulated in the deformed element is a sum of two components. The first component is caused by accumulation of the potential (accumulated, latent) energy U^e in the deformed volume of the material, and the second part of the energy is accumulated in the form of heat content U^T. Thus, based on the principle of energy superposition [13], we obtain

$$U_+ = U^e + U^T. \tag{4.1}$$

In accordance with the above-said, the destruction criterion is

$$U_+(t_*) = U_0 + U_1(t_*) = U_*, \tag{4.2}$$

where $U_0(T)$ is the initial value of the specific internal energy for $t = 0$, U_1 is the increment of the internal energy due to deformation, U_* is the critical value of the internal energy (a constant of a material), and t_* is the time before the moment of destruction. The increment $\Delta U_+ = \Delta U_1$ during the time Δt is composed of two parts:

$$\Delta U_+ = \Delta U^e + \Delta U^T (\Delta U^e = \sigma \Delta e^p + \sigma \Delta p, \quad \Delta U^T = \Delta U_1^T + \Delta U_2^T + \Delta U_3^T). \tag{4.3}$$

In contrast to [1, 3, 4], the increment of the potential energy ΔU^e is written here for the actual stress σ rather than for the nominal stress; the cyclic component σ_{a_0} affects the strains e^p and p through the damage parameter [relations (3.1) and (3.2)]. The quantities ΔU_1^T, ΔU_2^T, and ΔU_3^T are the increments of the heat content upon formation of plastic deformation, creep deformation, and cyclic loading. The next problem is to determine ΔU^T. A direct measurement of this quantity (and, moreover, its separation into the components ΔU_i^T) using calorimetry is a difficult problem even under laboratory conditions at a fixed temperature. Therefore, it is necessary to find some other methods for evaluating ΔU^T.

Some experimental data [12] allow us to adopt the following hypotheses: ΔU_1^T and ΔU_2^T are proportional to $\sigma \Delta e^p$ and $\sigma \Delta p$, respectively, and the value of ΔU_3^T at constant σ_0, σ_{a_0}, and f during the loading cycle is a certain part of the applied work of the actual amplitude stress during a half-cycle:

$$\Delta U_3^T = g_2(\sigma_0, \sigma_{a_0}, T, f) \, \frac{\sigma_a^2}{2E} \, N. \tag{4.4}$$

Taking into account (4.4), we transform (4.3) to

$$\Delta U_+ = \sigma \Delta e^p \Big(1 + \frac{\Delta U_1^T}{\sigma \Delta e^p}\Big) + \sigma \Delta p \Big(1 + \frac{\Delta U_2^T}{\sigma \Delta p}\Big) + g_2(\sigma_0, \sigma_{a_0}, T, f) \, \frac{\sigma_a^2}{2E} \, N. \tag{4.5}$$

Based on the hypotheses adopted and using the notation $1 + \Delta U_1^T/(\sigma \Delta e^p) = C(T)$ and $1 + \Delta U_2^T/(\sigma \Delta p) = D(\sigma_0, T)$, we rewrite relation (4.5) as

$$\Delta U_+ = C(T) \sigma \Delta e^p + D(\sigma_0, T) \sigma \Delta p + g_2(\sigma_0, \sigma_{a_0}, T, f) \, \frac{\sigma_a^2}{2E} \, \Delta N. \tag{4.6}$$

After integration of (4.6), using (4.2) we obtain

$$\int\limits_0^{t_*} C(T) \sigma \, de^p + \int\limits_0^{t_*} D(\sigma_0, T) \sigma \, dp + \int\limits_0^{t_*} g_2(\sigma_0, \sigma_{a_0}, T, f) \, \frac{\sigma_a^2}{2E} \, dN = U'(T). \tag{4.7}$$

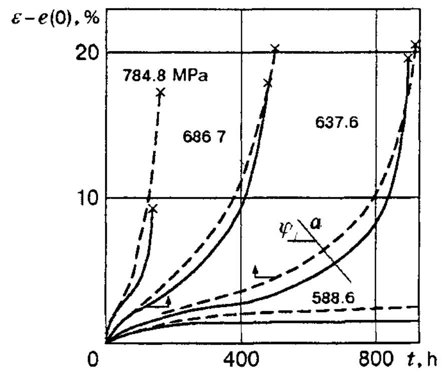

Fig. 1

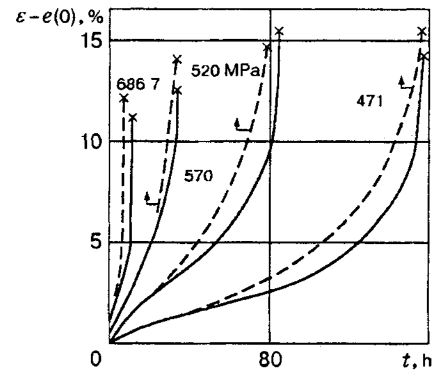

Fig. 2

Here $U'(T) = U_* - U_0(T)$. For a constant temperature T, relation (4.7) can be written as

$$\int_0^{t_*} \frac{\sigma\, de^p}{A_*^p(T)} + \int_0^{t_*} \frac{\sigma\, dp}{A_*^c(\sigma_0, T)} + \frac{1}{2E} \int_0^{t_*} \frac{\sigma_a^2\, dN}{A_*^y(\sigma_0, \sigma_{a_0}, T, f)} = 1, \tag{4.8}$$

where $A_*^p = U'(T)/C(T)$, $A_*^c = U'(T)/D(\sigma_0, T)$, and $A_*^y = U'(T)/g_2(\sigma_0, \sigma_{a_0}, T, f)$. In a special case, for $T = \text{const}$ and $f = \text{const}$, relation (4.8) becomes

$$\int_0^{t_*} \frac{\sigma\, de^p}{A_*^p} + \int_0^{t_*} \frac{\sigma\, dp}{A_*^c(\sigma_0)} + \frac{1}{2E} \int_0^{t_*} \frac{\sigma_a^2\, dN}{A_*^y(\sigma_0, \sigma_{a_0})} = 1. \tag{4.9}$$

Relation (4.8) or its special case (4.9) is the destruction criterion under a joint action of static and cyclic loads. Thus, the rheological model for the phenomenon described consists of the system of equations (2.1), (2.2), (3.1), (3.2), and (4.8) or (4.9).

Relations similar in structure to (4.8) and (4.9) but written for the nominal rather than actual stresses were considered for few-cycle fatigue by Romanov [14].

5. A technique for determining the parameters of the model proposed is described below. In the energetic approach, the following experimental data are used as the basic parameters:

— a diagram of material tension at a constant, rather high strain rate,

— a series of creep curves from the beginning of loading to the moment of destruction (marked by crosses in Figs. 1–4) for $\sigma_0 = \text{const}$, which are called the steady creep curves,

— a series of creep curves from the beginning of loading to the moment of destruction for $\sigma_0 = \text{const}$ and $\sigma_{a_0} = \text{const}$, which are called the steady cyclic creep curves.

Using the first two basic experiments and the technique described in [2], we determine all the parameters of Eqs. (2.1)–(2.5) and also the values of γ, α, A_*^p, and A_*^c; the value of A_*^c is approximated by $A_*^c = \alpha_*(\sigma_0)^{m_*}$. To determine $g_1(\sigma_0, \sigma_{a_0})$ and $A_*^y(\sigma_0, \sigma_{a_0})$, we use the steady cyclic creep curves. Numerical calculation of Eqs. (2.1), (2.2), (3.1), and (3.2) is performed for different values of g_1 until minimization of the functional characterizing the closeness of the calculated and experimental curves of inelastic deformation. The criterion of closeness of the calculated and experimental curve is the distance between them in a certain chosen direction φ $(0 \leqslant \varphi \leqslant \pi/2)$ (Fig. 1), and the measure of closeness is the dimensionless functional of the form

$$\sum_{j=1}^{M} \left\{ \left[\frac{p_j^{\text{calc}} - p_j^{\text{exp}}}{p_*} \right]^2 + \left[\frac{t_j^{\text{calc}} - t_j^{\text{exp}}}{t_*} \right]^2 \right\} \to \min, \tag{5.1}$$

where t_j^{calc} and p_j^{calc} are the calculated values and t_j^{exp} and p_j^{exp} are the experimental values of time and inelastic deformation corresponding to the points of intersection of the creep curves with the straight line a inclined at an angle φ to the t axis (Fig. 1), t_* and p_* are the experimental values of time and inelastic

TABLE 1

T, °C	λ_1, h^{-1}	a_1	b_1	c	n_1	m	α_1, MPa^{-1-m_1}	m_1	α_*, MPa^{-1-m_*}
650	0.022	$7.32 \cdot 10^{-4}$	$5.37 \cdot 10^{-3}$	$7.22 \cdot 10^{-7}$	3.29	14.3	$3.0 \cdot 10^{14}$	−6.09	174.4
750	0.2	$6.55 \cdot 10^{-4}$	$4.804 \cdot 10^{-3}$	$4.15 \cdot 10^{-5}$	3.76	8.9	$2.81 \cdot 10^{6}$	−3.3	81.1

TABLE 2

T, °C	σ_+, MPa	E, MPa	a, MPa^{-n}	n	γ_1, MPa^{-1}	A_*^p, $MJ \cdot m/m^3$	m_2
650	696.3	$1.79 \cdot 10^{-5}$	$8.614 \cdot 10^{-7}$	1.854	$1.776 \cdot 10^{-3}$	227.5	0
750	608.2	$1.70 \cdot 10^{-5}$	$5.102 \cdot 10^{-7}$	1.943	$1.623 \cdot 10^{-3}$	180.0	0

TABLE 3

T, °C	G_f, $MPa^{-1} \cdot h^{-1} \cdot Hz^{-1}$	α_f	n_f	A^y, $MJ \cdot m/m^3$	α_y	n_y	$\sigma_{a_0}^*$
650	$2.95 \cdot 10^{-5}$	−0.087	0.532	$9.58 \cdot 10^{9}$	−0.122	1.40	49
750	$2.71 \cdot 10^{-10}$	0.153	−1.240	$4.07 \cdot 10^{9}$	−0.168	0.82	49

deformation corresponding to the point of sample destruction, and M is the number of points used for minimization of functional (5.1).

In a special case, criterion (5.1) includes traditional methods for determining the closeness of the curves on the basis of deformation. For $\varphi = \pi/2$, we have the method for determining the closeness on the basis of deformation and for $\varphi = 0$ on the basis of the time of reaching a given value of inelastic strain. At the same time, criterion (5.1) is free of their drawbacks. For example, the criterion of closeness based on deformation is inapplicable at the third stage, and the closeness based on time can yield significant errors for materials with a low velocity at the second stage of creep.

Determining g_1 for given σ_{a_0} and σ_0, we continue numerical calculations using the model until the calculated inelastic strain reaches the experimental value at the moment of destruction and find the value of A_*^y from the relation

$$A_*^y = \frac{1}{2E} \int_0^{t_*} \sigma_a^2 f \, dt \Big/ \left(1 - \int_0^{t_*} \frac{\sigma \, de^p}{A_*^p} - \int_0^{t_*} \frac{\sigma \, dp}{A_*^c(\sigma_0)}\right). \tag{5.2}$$

After determining g_1 and A_*^y for several constant σ_{a_0} and σ_0, we construct a two-dimensional approximation of these quantities. An analysis of experimental data shows that we can use the following expressions for approximation:

$$g_1(\sigma_0, \sigma_{a_0}) = G_f \exp\left[\alpha_f \frac{\sigma_0}{\sigma_*}\right] \left(\frac{\sigma_{a_0}}{\sigma_{a_0}^*}\right)^{n_f}, \qquad A_*^y = A^y \exp\left[\alpha_y \frac{\sigma_0}{\sigma_*}\right] \left(\frac{\sigma_{a_0}}{\sigma_{a_0}^*}\right)^{n_y}. \tag{5.3}$$

6. The validity of the equations proposed was experimentally verified using the ÉP742 material for $T = 650$ and 750°C, loading frequency $f = 50$ Hz, and a sine-shaped cyclic component of stress. Radchenko [2] performed a detailed verification of Eqs. (2.1)–(2.5) and destruction criterion (4.9) under conditions of quasi-static creep (cyclic component $\sigma_{a_0} = 0$) for various regimes of loading σ_0 for this material.

The initial information for determining the parameters of model (2.1)–(2.5) and (4.9) were the experimental curves of steady creep shown by solid curves in Fig. 1 for $T = 650$°C and Fig. 2 for $T = 750$°C, where $e(0)$ is the value of elastic strain at the moment of load application at $t = 0$. The values of the parameters are given in Table 1 for creep deformation ($\sigma_* = 490.5$ MPa, $K = 1$, and $m_* = 0$ for both values of temperature)

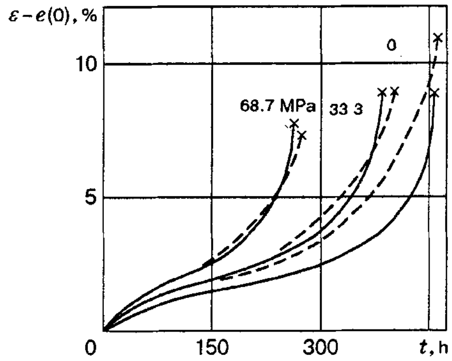

Fig. 3

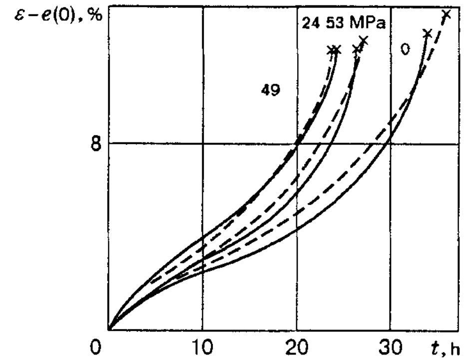

Fig. 4

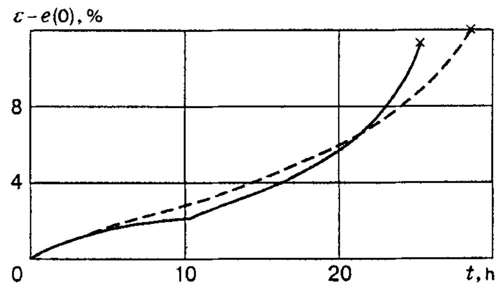

Fig. 5

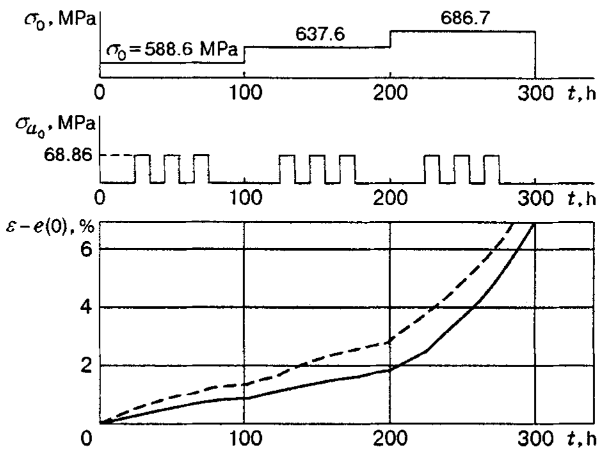

Fig. 6

and in Table 2 for plastic deformation. The dashed curves in Figs. 1 and 2 correspond to the calculation performed using model (2.1)–(2.5), (4.9). The arrows indicate the beginning of plastic deformation. At this time, the actual stress σ is greater than the limit of proportionality σ_+ due to damage accumulation, though the inequality $\sigma_0 < \sigma_+$ was valid at the initial time $t = 0$.

The data on cyclic creep of the ÉP742 alloy for model (2.1), (2.2), (3.1), (3.2), (4.9) calculated using the technique described in Sec. 5 are listed in Table 3. As an example, Figs. 3 and 4 show experimental (solid curves) and calculated (dashed curves) values of inelastic strain under steady cyclic creep. The curves in Fig. 3 correspond to $T = 650°$C and $\sigma_0 = 686.7$ MPa, those in Fig. 4 to $T = 750°$C and $\sigma_0 = 569$ MPa, and the values of the cyclic component σ_{a_0} for each curve are given by numbers.

Figures 5 and 6 show experimental (solid curves) and calculated (dashed curves) values of inelastic rheological strain in a complex unsteady regime for both static and cyclic components of loading. The curves in Fig. 5 correspond to $T = 750°$C, the quantity $\sigma_0 = 570$ MPa was constant during the experiment, and the cyclic component was $\sigma_{a_0} = 0$ for $t \in [0, 10]$ and $\sigma_{a_0} = 49$ MPa for $t \geqslant 10$. Figure 6 ($T = 650°$C) shows variation of the components σ_0 and σ_{a_0} for complex loading programs given at the top of the figure.

The examples presented demonstrate good agreement between the calculated and experimental data. The model proposed describes two basic effects of cyclic creep in the region considered [5]: acceleration of the creep process and reduction of the accumulated inelastic strain at the moment of destruction as compared to quasi-static creep.

REFERENCES

1. O. V. Sosnin, B. V. Gorev, and A. F. Nikitenko, *Energetic Variant of Creep Theory* [in Russian], Inst. of Hydrodynamics, Sib. Div., USSR Acad. of Sci., Novosibirsk (1986).
2. V. P. Radchenko, "Energetic variant of uniaxial creep theory and long-term strength," *Prikl. Mekh. Tekh. Fiz.*, No. 4, 172–179 (1991).
3. V. V. Fedorov, "Thermodynamic concepts of strength and destruction of a solid body," *Probl. Prochn.*, No. 11, 32–34 (1971).
4. D. A. Kiyalbaev and A. I. Chudnovskii, "Destruction of deformable bodies," *Prikl. Mekh. Tekh. Fiz.*, No. 3, 105–110 (1970).
5. V. P. Golub, "Some effects of creep under cyclic loading," *Probl. Prochn.*, No. 5, 20–24 (1987).
6. A. M. Lokoshchenko, E. A. Myakotin, and S. A. Shesterikov, "Effect of small vibrations on creep," *Probl. Prochn.*, No. 5, 50–54 (1985).
7. Yu. N. Rabotnov, *Creep Problems in Structural Members*, North-Holland, Amsterdam (1969).
8. V. T. Troshchenko, *Fatigue and Inelasticity of Metals* [in Russian], Naukova Dumka, Kiev (1971).
9. G. I. Barenblatt, Yu. I. Kozyrev, N. L. Malinin, et al., "Vibrocreep of polymeric materials," *Prikl. Mekh. Tekh. Fiz.*, No. 5, 68–75 (1965).
10. O. V. Sosnin, B. V. Gorev, and V. V. Rubanov, "Creep of cyclically loaded elements of constructions," *Probl. Prochn.*, No. 10, 66–69 (1977).
11. Ya. I. Tsimbalistyi, I. A. Troyan, and O. I. Marusin, "Vibrocreep of the ÉI437B alloy at normal and high temperatures," *Probl. Prochn.*, No. 11, 30–35 (1975).
12. V. V. Fedorov, *Kinetics of Damage and Destruction of Solids* [in Russian], Fan, Tashkent (1985).
13. M. Planck, *Principle of Energy Conservation* [Russian translation], GONTI, Moscow–Leningrad (1938).
14. A. N. Romanov, "Energetic criteria of destruction under few-cycle loading," *Probl. Prochn.*, No. 1, 4–13 (1974).